

## Derivation of Zoo3 DD Contribution

Definition of Unavailability

$$Q = \text{Unavailability} = \frac{\text{Downtime}}{\text{Total Time}}$$

For a mean cycle of a repairable system...

$$\text{Downtime} = \text{MTTR} = \text{Mean Time to Restore} \\ (\text{including detection \& repair})$$

$$\text{Total Time} = \text{MTBF} = \text{Mean Time Between Failures}$$

$$\text{MTBF} = \text{MTTR} + \text{MTTF} \quad (\text{Mean Time to Failure})$$

Therefore:

$$Q = \frac{\text{Downtime}}{\text{Total Time}} = \frac{\text{MTTR}}{\text{MTTF} + \text{MTTR}}$$

If one assumes that MTTR is negligibly small in comparison to MTTF...

$$\text{MTTR} \ll \text{MTTF} \Rightarrow \text{MTTF} + \text{MTTR} \approx \text{MTTF}$$

so it follows...

$$Q \approx \frac{\text{MTTR}}{\text{MTTF}}$$

For a constant failure rate system

$$\lambda = \text{failure rate} = 1 / \text{MTTF}$$

so it follows...

$$Q \approx \lambda * \text{MTTR}$$

## Derivation of 2003 DD Contribution

For this case we are concerned with detected dangerous failures which are the repairable failures that make a SIF unavailable.

$$\lambda = \lambda_{DD}$$

So

$$Q_{DD} \approx \lambda_{DD} \text{MTTR}$$

also, MTTR can be decomposed into the time period required to perform the repair, MRT - Mean Repair Time and the time required to detect the failure, half of the diagnostic test interval  $T_{IA}/2$

$$\text{MTTR} = \text{MRT} + T_{IA}/2$$

$$\therefore Q_{DD} = \lambda_{DD} (\text{MRT} + T_{IA}/2) \quad (1)$$

This is then the unavailability due to diagnosed detected failures for a single component, i.e. 1ook.

For 2003, where components are A, B, and C

$$Q_{2003} = (Q_{A \text{ and } B}) \text{ OR } (Q_{A \text{ and } C}) \text{ OR } (Q_{B \text{ and } C})$$

$$\begin{aligned} Q_{A \text{ and } B} &= \lambda_{DD} (\text{MRT} + T_{IA}/2) * \lambda_{DD} (\text{MRT} + T_{IA}/2) \\ &= \lambda_{DD}^2 (\text{MRT} + T_{IA}/2)^2 \quad [\text{Probability Multiplication}] \end{aligned}$$

$$\begin{aligned} Q_{2003, DD} &= \lambda_{DD}^2 (\text{MRT} + T_{IA}/2)^2 + \lambda_{DD}^2 (\text{MRT} + T_{IA}/2)^2 \\ &\quad + \lambda_{DD}^2 (\text{MRT} + T_{IA}/2)^2 \end{aligned}$$

$$= 3 \lambda_{DD}^2 (\text{MRT} + T_{IA}/2)^2$$